

Testing Functional Forms in Energy Modeling: An Application of the Bayesian Approach to U.S. Electricity Demand

Ni Xiao

Graduate School of Business, University of Chicago

Jay Zarnikau*

Frontier Associates, Austin and the University of Texas at Austin

Paul Damien

McCombs School of Business, University of Texas at Austin

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Abstract

In the estimation of demand functions for energy resources, linear, log-linear, translog, or AIDS functional forms are commonly assumed. It is frequently debated which is the “best” functional forms to employ in order to accurately represent the underlying relationships between the consumption of various energy resources and explanatory variables such as energy prices, weather variables, income, and other factors.

In an example analysis of residential energy demand employing cross-sectional household-level data, we find that the AIDS model is slightly better than the translog model, which is superior to the log-linear model, and which in turn is better than the linear model. This is in sharp contrast to previous findings in this context.

Keywords: Functional form, energy demand, Bayesian statistics.

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1 Introduction

Parametric econometric models of energy demand are commonly used to predict future energy needs under alternative economic and policy scenarios. Elasticity estimates from such models are used to analyze how changes in energy prices, tax changes, income, weather, and other factors might affect the demand for various energy resources.

*Corresponding author: Jay Zarnikau, Frontier Associates, 4131 Spicewood Springs Road, Suite O-3, Austin, TX 78759, USA. E-mail address: jayz@frontierassoc.com. Telephone: (512) 372-8778

The functional forms commonly assumed in parametric energy demand models include linear functional forms, log-linear forms, and translog models. In linear models, various explanatory variables are assumed to affect energy demand in a simple linear fashion. In log-linear models, the dependent and explanatory variables are transformed into natural logarithms and then regressed. Elasticities may then be readily obtained from the estimated coefficients. Linear and log-linear models are sometimes referred to as ad hoc models, to acknowledge their limited theoretical foundation. In contrast, translog models have some basis in microeconomic theory and are popular in the academic literature. Examples of translog models of energy demand may be found in Uri (1982) and Watkins (1992). A variety of other models are found in the academic literature, but less often used by practitioners, including Almost Ideal Demand Systems (AIDS), Symmetric Generalized McFadden (SGM), and Generalized Leontief (GL) forms.

In an earlier article, Zarnikau (2003) used frequentist non-parametric bootstrapping methods to compare linear, log-linear, and translog share equation functional forms against a nonparametric function. In this paper, we repeat this exercise using the same example application and dataset, but with an alternative Bayesian approach. In particular, we introduce a new model choice criterion, which when applied to these data shows that the AIDS and translog functional form outperform the other specifications considered in this paper. This is in sharp contrast to earlier work on this topic.

2 Example Application

A researcher faced with the task of estimating the heating season (winter) household demand for electricity using cross-sectional data and with limited information about appliance stocks, the housing stock, and other end-use factors, might consider the following functional forms:

- Linear:

$$KWH_i = a_{E1} + b_{E1} \cdot PE_i + b_{N1} \cdot PN_i + b_{I1} \cdot INC_i + b_{HDD1} \cdot HDD_i \quad (1)$$

- Log-Linear:

$$\log(KWH)_i = a_{E2} + b_{E2} \cdot \log(PE_i) + b_{N2} \cdot \log(PN_i) + b_{I2} \cdot \log(INC_i) + b_{HDD2} \cdot \log(HDD_i) \quad (2)$$

- Translog Share Equation:

$$SE_i = a_{E3} + b_{E3} \cdot \log(PE_i) + b_{N3} \cdot \log(PN_i) + b_{I3} \cdot \log(INC_i) + b_{HDD3} \cdot \log(HDD_i) \quad (3)$$

- Linear approximate formulation of the Almost Ideal Demand System (AIDS):

$$SE_i = a_{E4} + b_{E4} \cdot \log(PENORM_i) + b_{N4} \cdot \log(PNNORM_i) + b_{X4} \cdot \log\left(\frac{X_i}{P_i}\right) + b_{HDD4} \cdot HDD_i \quad (4)$$

In the AIDS model, $PENORM_i$ is the mean-scaled price of electricity; $PNNORM_i$ is the mean-scaled price of natural gas; X_i is household i 's total expenditure on electricity and

nature gas; and P_i is the price index, which is approximated by $\log(P_i) = SE_i \cdot \log(PE_i) + (1 - SE_i) \cdot \log(PN_i)$, as suggested in Deaton and Muellbauer (1980). Additionally, in these equations, KWH_i is the household's total electricity consumption. SE_i and SN_i are the share of household i 's total expenditures on energy resources which were spent on electricity and natural gas, respectively. PE_i represents the price of electricity faced by the household; while PN_i is the price of natural gas, a common alternative energy source for space heating, cooking, and water heating. INC_i is the household's income. HDD represents heating degree days, an indicator of weather and space heating needs. The translog and AIDS models can be estimated as single electricity share equations, as in Equation (3) and Equation (4), since only two energy resources are being considered and one expenditure share equation must be dropped when a set of share equations is estimated¹.

Ideally, a number of other variables should be included in these models to both enhance explanatory power and to overcome concerns about the validity of weak separability assumptions. However, seldom do practitioners have available to them data necessary to fully test and implement the specifications suggested by theorists. Thus, these models reflect the types of data to which practitioners normally have access, rather than data they would ideally prefer to access.

3 Specification Testing

While regression test statistics provide some indication of which, if any, of these parametric functional forms are valid, more formal tests can be constructed. One class of specification tests using non-parametric techniques was developed in the classical or frequentist tradition. These tests compare parametric models to nonparametric kernel regressions. Examples can be found in Härdle and Mammen (1993) and Zheng (1996).

Another set of tests has been developed in the Bayesian tradition. Raftery and Richardson (1993) explore model selection in epidemiological applications using Bayes factors. The application of Markov chain Monte Carlo (MCMC) approaches in Bayesian analysis can be found in Gelman et al. (1998). We use MCMC methods in the analysis of data.

This paper employs a new approach, the Deviance Information Criterion (DIC) developed by Spiegelhalter et al. (2000). These authors show that the DIC is better than other model selection criteria. The DIC approach has an advantage over a Bayes factor approach in applications such as the one considered in this paper. There are difficulties in applying a simple Bayes factor approach when the dependent variables used in different models are different or transformations of each other. In the three models compared here, the three dependent variables are electricity consumption (in level form), the natural logarithm of electricity consumption, and the share of expenditures on electricity consumption. The DIC approach provides results which are invariant to the transformations applied to the dependent variable. This paper tests whether the linear, log-linear, AIDS or translog parametric functional forms provide reasonable representations of the "true" functional relationship

¹For the translog model, the coefficients on the price terms in the natural gas expenditure share equation can be derived from the b_E and b_N parameters estimated from the electricity share equation, if certain microeconomic relationships are assumed to hold. Typically, when energy demand functions are estimated, the share equations (rather than the cost function) are directly estimated.

between the U.S. household-level demand for electricity and various explanatory variables using the DIC. Data and modeling techniques are described in the following sections.

4 Dataset

In this analysis, we use the dataset previously described in Zarnikau (2003). The rationale for the many assumptions that went into the selection of the data can be found in Zarnikau (2003). Here we merely note certain key features.

First, household-level cross-sectional data for 5,235 households was obtained from the UC-Berkeley web site. Second, household electricity cost and natural gas cost for the first quarter of 1994 was obtained from the Consumer Expenditure (CE) survey database compiled by the U.S. Bureau of Labor Statistics (U.S. BLS). Third, the household's state of residence, total income, and certain appliance saturation information was extracted from the CE's Consumer Unit and Characteristics and Income File (FMLY). Fourth, average price of electricity and natural gas to residential energy consumers in each state in 1994 was obtained from the U.S. Department of Energy (U.S. DOE) Energy Information Agency's State Energy Price and Expenditure Report database. Fifth, weather data were obtained from the National Climatic Data Center for the first quarter of 1994. Sixth, quarterly data were used in the analysis because, as noted by Zarnikau (2003), smaller time intervals provide more meaningful results for this sector with weather-sensitive energy demand. Sixth, after eliminating missing values and zero natural gas use records, resulted in 1341 complete observations. Lastly, electricity and gas prices can be double in certain states such as New York, California, Hawaii than in low cost states in the Northwest and Southeast.

5 Modeling Approach

The paper adopts a Bayesian approach to model uncertainty in the data. Recent advances in computer simulations have made it easy to implement Bayesian analysis in a variety of contexts; see, for example, Gelman et al. (1998). Throughout, we employ non-informative priors, thus letting the underlying model and data dictate the analysis and results.

5.1 Bayesian Approach

The four models (1), (2), (3), and (4) can be written as the following model except that y_i and \underline{x} denote different forms of variables in the original models; see, for example, Zellner (1971).

$$y_i = \underline{\beta}^T \underline{x}_i + \varepsilon_i,$$

where $i = 1, 2, \dots, n$, $\underline{\beta} = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)^T$ and $\underline{x}_i = (x_{0i}, x_{1i}, x_{2i}, x_{3i}, x_{4i})$ with $x_{0i} = 1$. Assume the errors ε_i are mutually independent and normal distributed with mean 0 and unknown variance σ^2 . Then it follows that

$$y_i \sim \text{Normal}(\underline{\beta}^T \underline{x}_i, \sigma^2),$$

and thus, the likelihood function can be written as:

$$L(\underline{y}|\underline{\beta}, \sigma) = (2\pi)^{-\frac{n}{2}} \sigma^{-n} \exp \left\{ -\frac{\sum_{i=1}^n (y_i - \underline{\beta}^T \underline{x}_i)^2}{2\sigma^2} \right\}.$$

If we take $\frac{1}{\sigma}$ as the joint prior on $(\underline{\beta}, \sigma)$, then the joint posterior distribution is obtained as

$$f(\underline{\beta}, \sigma | \underline{y}) \propto \sigma^{-n-1} \exp \left\{ -\frac{\sum_{i=1}^n (y_i - \underline{\beta}^T \underline{x}_i)^2}{2\sigma^2} \right\}. \quad (5)$$

The conditional posterior distribution of the parameter β_j , ($j = 0, 1, 2, 3, 4$), given σ , \underline{y} and other β_j 's, is the exponential of a quadratic form in β_j . Hence, it is normal:

$$\beta_j | (\underline{\beta}_{(\setminus j)}, \sigma, \underline{y}) \sim \text{Normal} \left(\frac{\sum_{i=1}^n x_{ji} (y_i - \underline{\beta}_{(\setminus j)}^T \underline{x}_{(\setminus j)i})}{\sum_{i=1}^n x_{ji}^2}, \frac{\sigma^2}{\sum_{i=1}^n x_{ji}^2} \right)$$

for $j = 0, 1, 2, 3, 4$, where $\underline{\beta}_{(\setminus j)}$ denotes the vector consisting of all entries β_k except for β_j (with $k \neq j$), and similarly $\underline{x}_{(\setminus j)i}$ denotes the vector consisting of all entries x_{ki} with $k \neq j$. Since $x_{0i} = 1$, the conditional posterior distribution of the parameter β_0 can be simplified as

$$\beta_0 | (\underline{\beta}_{(\setminus 0)}, \sigma, \underline{y}) \sim \text{Normal} \left(\frac{\sum_{i=1}^n (y_i - \underline{\beta}_{(\setminus 0)}^T \underline{x}_{(\setminus 0)i})}{n}, \frac{\sigma^2}{n} \right).$$

In addition, from (5) we can obtain the conditional posterior distribution of $\frac{1}{\sigma^2}$, given $\underline{\beta}$ and \underline{y} as

$$\frac{1}{\sigma^2} | (\underline{\beta}, \underline{y}) \sim \text{Gamma} \left(\frac{n}{2}, \frac{\sum_{i=1}^n (y_i - \underline{\beta}^T \underline{x}_i)^2}{2} \right).$$

Using the conditional distributions shown above, we developed a Markov chain Monte Carlo algorithm (Smith and Roberts, 1993) to simulate the marginal distribution of each β_i and σ .

5.2 Deviance Information Criterion

To measure model complexity and fit, we used the Deviance Information Criterion DIC (Spiegelhalter et al., 2002). The DIC is a very nice metric for model choice because, unlike other approaches, it is decomposed into two terms: one accounts for model fit and the second accounts for model size. As Spiegelhalter et al. show, well-known model selection metrics like the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are limiting cases of the DIC. Models with smaller values of DIC are preferred to ones with larger DIC values. This is also the basis for other model selection criteria such as the AIC and BIC.

To calculate the DIC using the Markov chain Monte Carlo samples from the posterior distributions of the parameters in the regression models, first, let's define

$$D(\theta) = -2LL(\underline{y}|\theta) + 2\log(f(\underline{y})),$$

where θ denotes the parameters in likelihood function and LL is Log-Likelihood of the data evaluated at θ . Then, the DIC is defined as following:

$$DIC = 2\overline{D(\theta)} - D(\bar{\theta}),$$

where $\overline{D(\theta)}$ is the average of $D(\theta)$ and $D(\bar{\theta})$ is $D(\theta)$ computed at the average values of θ . For our models, $\theta = (\underline{\beta}, \sigma)$ and $f(\underline{y})$ is set to equal 1. The latter choice is recommended by Spiegelhalter et al. Since each model has a different likelihood, we compute them separately as follows. (A comment on notation: for ease in presenting the results in Table 1, below we omit the multiple subscripts that we first introduced while describing the four models, noting there is no ambiguity in the ensuing depiction of the four models.)

- Model 1: Linear Model

$$y_i = \beta_0 + \beta_1 \cdot x_{1i} + \beta_2 \cdot x_{2i} + \beta_3 \cdot x_{3i} + \beta_4 \cdot x_{4i} + \varepsilon_i;$$

We know that

$$y_i \sim Normal(\underline{\beta}^T \underline{x}_i, \sigma^2),$$

and therefore the likelihood function can be written as:

$$L(\underline{y}|\underline{\beta}, \sigma) = (2\pi)^{-\frac{n}{2}} \sigma^{-n} \exp \left\{ -\frac{\sum_{i=1}^n (y_i - \underline{\beta}^T \underline{x}_i)^2}{2\sigma^2} \right\}.$$

Thus,

$$D(\theta) = n \log 2\pi + 2n \log \sigma + \frac{1}{\sigma^2} \sum_{i=1}^n (y_i - \underline{\beta}^T \underline{x}_i)^2.$$

- Model 2: Log-Linear Model

$$\log(y_i) = \beta_0 + \beta_1 \cdot \log(x_{1i}) + \beta_2 \cdot \log(x_{2i}) + \beta_3 \cdot \log(x_{3i}) + \beta_4 \cdot \log(x_{4i}) + \varepsilon_i;$$

Since

$$\log(y_i) \sim Normal(\underline{\beta}^T \underline{\log(x)}_i, \sigma^2),$$

therefore the likelihood function can be written as:

$$L(\underline{y}|\underline{\beta}, \sigma) = (2\pi)^{-\frac{n}{2}} \sigma^{-n} \exp \left\{ -\frac{\sum_{i=1}^n (\log(y_i) - \underline{\beta}^T \underline{\log(x)}_i)^2}{2\sigma^2} \right\} \prod_{i=1}^n y_i^{-1}.$$

Thus,

$$D(\theta) = n \log 2\pi + 2n \log \sigma + \frac{1}{\sigma^2} \sum_{i=1}^n (\log(y_i) - \underline{\beta}^T \underline{\log(x)}_i)^2 + 2 \sum_{i=1}^n \log(y_i).$$

- Model 3: Translog Share Model

$$y_i a_i = \beta_0 + \beta_1 \cdot \log(x_{1i}) + \beta_2 \cdot \log(x_{2i}) + \beta_3 \cdot \log(x_{3i}) + \beta_4 \cdot \log(x_{4i}) + \varepsilon_i;$$

where a_i denotes ratio of the price of electricity and the total expenditures on energy resources of household i . So $y_i a_i$ is the share of household i 's total expenditures on energy resources which were spent on electricity. Now we know that

$$y_i a_i \sim \text{Normal}(\underline{\beta}^T \underline{\log}(x)_i, \sigma^2);$$

hence, the likelihood function can be written as:

$$L(\underline{y}|\underline{\beta}, \sigma) = (2\pi)^{-\frac{n}{2}} \sigma^{-n} \exp \left\{ -\frac{\sum_{i=1}^n (y_i a_i - \underline{\beta}^T \underline{\log}(x)_i)^2}{2\sigma^2} \right\} \prod_{i=1}^n a_i.$$

Thus,

$$D(\theta) = n \log 2\pi + 2n \log \sigma + \frac{1}{\sigma^2} \sum_{i=1}^n (y_i a_i - \underline{\beta}^T \underline{\log}(x)_i)^2 - 2 \sum_{i=1}^n \log(a_i).$$

- Model 4: Almost Ideal Demand System Model

$$y_i a_i = \beta_0 + \beta_1 \cdot \log(x_{1i}) + \beta_2 \cdot \log(x_{2i}) + \beta_3 \cdot \log(x_{3i}) + \beta_4 \cdot x_{4i} + \varepsilon_i;$$

where a_i denotes ratio of the price of electricity and the total expenditures on energy resources of household i . So $y_i a_i$ is the share of household i 's total expenditures on energy resources which were spent on electricity. Now we know that

$$y_i a_i \sim \text{Normal}(\underline{\beta}^T \underline{X}_i, \sigma^2),$$

where $\underline{X}_i = (1, \log(x_{1i}), \log(x_{2i}), \log(x_{3i}), x_{4i})^T$; hence, the likelihood function can be written as:

$$L(\underline{y}|\underline{\beta}, \sigma) = (2\pi)^{-\frac{n}{2}} \sigma^{-n} \exp \left\{ -\frac{\sum_{i=1}^n (y_i a_i - \underline{\beta}^T \underline{X}_i)^2}{2\sigma^2} \right\} \prod_{i=1}^n a_i.$$

Thus,

$$D(\theta) = n \log 2\pi + 2n \log \sigma + \frac{1}{\sigma^2} \sum_{i=1}^n (y_i a_i - \underline{\beta}^T \underline{X}_i)^2 - 2 \sum_{i=1}^n \log(a_i).$$

6 Numerical Results

We ran the Markov chain Monte Carlo algorithm for 5000 iterations. Following Smith and Roberts (1993), standard diagnostic methods such as percentile monitoring and quantile plots indicated that convergence was attained in less than 3000 iterations. Thus using a "burn-in" of 3000 simulations, the DIC values were calculated using the last 2000 simulated

values. All analysis was programmed in *R*. Least square estimates for the coefficients were used as starting values, although it should be noted that the results were not sensitive to starting values because of the non-informative priors, and the fact that standard MCMC convergence diagnostics (such as Q-Q plots) indicated stability in the posterior estimates based on the above-mentioned iterations of the Gibbs sampler.

The simulated results and DIC value are summarized in Table 1 and Table 2. Except for β_2 in the linear model, and β_4 in the linear and log-linear models, from Table 1, it is clear that all the other independent variables are statistically significant in all the models. This is seen by noting that the ratios of the regression parameter estimates to their standard deviations (sd) are fairly small, or you can also observe this by noting that the 95% credible intervals for these regression parameters do not include zero.

Table 2 lists the three DIC values for the four models. It is clear that the AIDS and translog specifications outperform the log-linear and linear specifications, and do so convincingly. This is in sharp contrast to results reported elsewhere, for example, Zarnikau (2003); see next section for details on this point.

	$E[\beta_0 y]$	$sd[\beta_0 y]$	95% CI	$E[\beta_1 y]$	$sd[\beta_1 y]$	95% CI
Linear	555.9	72.61	(423.2, 712.4)	-11.85	2.164	(-16.18, -7.683)
Log-Linear	6.289	0.374	(5.584, 7.096)	-0.851	0.083	(-0.974, -0.698)
Translog	0.773	0.062	(0.647, 0.898)	0.185	0.017	(0.149, 0.221)
AIDS	0.843	0.033	(0.779, 0.906)	0.123	0.025	(0.072, 0.173)
	$E[\beta_2 y]$	$sd[\beta_2 y]$	95% CI	$E[\beta_3 y]$	$sd[\beta_3 y]$	95% CI
Linear	9.768	7.957	(-6.332, 24.67)	0.0026	0.0003	(0.0020, 0.0033)
Log-Linear	0.249	0.119	(0.027, 0.459)	0.1328	0.0171	(0.1007, 0.1657)
Translog	-0.194	0.031	(-0.243, -0.134)	0.0119	0.0033	(0.0059, 0.0185)
AIDS	-0.161	0.031	(-0.219, -0.100)	-0.0349	0.0063	(-0.0468, -0.0224)
	$E[\beta_4 y]$	$sd[\beta_4 y]$	95% CI	$E[\sigma y]$	$sd[\sigma y]$	95% CI
Linear	-0.009	0.012	(-0.033, 0.013)	312.2	7.299	(298.7, 327.1)
Log-Linear	0.047	0.027	(-0.001, 0.101)	0.682	0.016	(0.653, 0.714)
Translog	-0.088	0.005	(-0.099, -0.077)	0.156	0.004	(0.149, 0.163)
AIDS	-0.00008	0.000006	(-0.00009, -0.00007)	0.148	0.003	(0.141, 0.154)

Table 1: Simulated Results for Parameters Using Gibbs Sampling

	Linear Model	Log-Linear Model	Translog Share Model	AIDS
DIC	13615.55	12748.97	11523.69	11413.12

Table 2: DIC Values for Linear Model, Log-Linear Model and Translog Share Model Using Gibbs Sample Values for $\theta = (\underline{\beta}, \sigma)$

7 Discussion

In this paper, a new model selection criterion, the DIC, based on Spiegelhalter et al. (2002) is introduced. In the example analysis of US residential energy demand using cross-sectional household-level data, a comparison of the DIC for four well-known models shows that the AIDS and translog models are superior to a log-linear model, which in turn is better than the linear model.

Zarnikau (2003), using frequentist non-parametric techniques and this same dataset, found that none of the three functional forms tested could provide a satisfactory representation of the "true" underlying functional form. Yet a simple linear representation was found to be superior to the other two alternatives in the sense that the predicted values from a linear functional form were within a 95% confidence interval around a non-parametric representation of the model with a 29% frequency. Log-linear and translog parametric models provided predicted values that were within 95% confidence intervals constructed around non-parametric alternatives with lesser frequencies of 18% and 10%, respectively. Thus, it is interesting that the Bayesian approach, pursued here, provides an opposite ranking of the validity of these three models in this paper. Additionally, the AIDS model was also studied in this paper, which comes in first place using the DIC.

These findings highlight what is already obvious to many energy economists, namely that model selection remains a very difficult task and different model selection criteria may steer one towards different model choices.

References

- [1] Cosidine, T., 1989, "Separability, Functional Form and Regulatory Policy in Models of Interfuel Substitution", *Energy Economics*, 82-94.
- [2] Deaton, A.S. and Muellbauer, J.N., 1980, "An Almost Ideal Demand System," *American Economic Review*, 83, 570-597.
- [3] Gelman, A., Carlin, J.B., Stern, H.S., and Rubin, D.B., 1998, *Bayesian Data Analysis*, Chapman-Hall, New York.
- [4] Härdle, W. and E. Mammen, 1993, "Comparing Nonparametric Versus Parametric Regression Fits", *Annals of Statistics*, Vol. 21, No. 4, 1926-1947.
- [5] Smith, A.F.M. and Roberts, G.O., 1993, "Bayesian Computation via the Gibbs Sampler and Related Markov Chain Monte Carlo Methods (with discussion)", *Journal of the Royal Statistical Society*, B, 55, 3-24.
- [6] Spiegelhalter, D.J., Best, N.G., Carlin, B.P., Linde, A., 2002, "Bayesian Measures of Model Complexity and Fit (with discussion)", *Journal of the Royal Statistical Society*, B, 64, 583-639.
- [7] Uri, Noel, 1982, "The Industrial Demand for Energy", *Resources and Energy*, 4, 27-57.
- [8] Watkins, G.C., 1992, "The Econometric Analysis of Energy Demand: Perspectives of a Practitioner", in *Energy Demand: Evidence and Expectations*, ed. by David Hawdon, Surrey University Press.
- [9] Zarnikau, J., 2003, "Functional Forms in Energy Demand Modeling", *Energy Economics*, 25(6), November, 603-613.
- [10] Zellner, A., 1971, *An Introduction to Bayesian Inference in Econometrics*, Wiley, New York.
- [11] Zheng, J., 1996, "A Consistent Test of Functional Form via Nonparametric Estimation Functions", *Journal of Econometrics*, 75, 263-290.